Mat 2377: Quiz #6

July 22, 2016

Answer all the questions first in one 45 minute sitting. Then check your answers against the solutions given below.

1. Suppose that we have a binomial random variable Y with parameters $n = 192, p$. We would like to test the hypothesis

$$H_0 : p = 0.75, H_1 : p > 0.75$$

We reject the null hypothesis if and only if $Y \geq 152$. Use the normal approximation to calculate

a) $\alpha = P(Y \geq 152 | p = 0.75)$

b) $\beta = P(Y < 152 | p = 0.80)$

2. A machine shop manufactures toggle levers during day and night shifts. Let p_1, p_2 be the proportion of defective levers in the day and night shifts respectively. Test the hypothesis $H_0 : p_1 = p_2$ against a two sided alternative based on samples of 1000 levers from each shift.

a) Define the test statistic and a critical region that has a 5% significance level

b) If we observe 37 and 53 defectives respectively forthe day and night shifts, what is your conclusion?

3. A paper mill believes that the recently adopted measures have reduced

the oxygen-consuming power of their wastes from a previouos average of $\mu = 500$(measured in permangamate inparts per million). Test the hypotheis that the mean is 500 against the alternative that it is less than 500. Twenty five readings are taken and the sample mean is 308.8 and the sample standard deviation is 115.15.

a) Test the claim at the 1% level.

b) What is the p-value?

4. A psychologist claims that the variance of IQ scores for college students is $\sigma^2 = 100$.Test this claim against the alternative that the variance is not equal to 100 at the 5% significance level. A sample of 23 yielded a sample variance of 147.82.

5. Suppose that we wish to compare the weight of calcium in standard cement and cement doped with lead. Reduced levels of calcium would imply that water would attack various locations in the cement structure. Assuming normal distributions, test the hypothesis that $\mu_X = \mu_Y$ against the alternative that they are different using a 5% significance level and assuming a common variance for the two distributions.

Section	sample mean	sample variance	sample size
Standard cement (X)	90	25	10
lead doped cement (Y)	87	16	15

6. A vendor of milk products produces and sells low-fat milk to a company that uses it to produce baby formula. In order to determine the fat content of the milk, both the company and the vendor take a sample fromeach lot and test it for the percentage of fat content. Ten sets of paired test results are obtained. Test the null hypothesis that there is no difference in the mean percentage fat content against the alternative that the means differ. Use a 5% significance level.

Lot number	Company test results (X)	Vendor test results (Y)
1	0.50	0.79
2	0.58	0.71
3	0.90	0.82
4	1.17	0.82
5	1.14	0.73
6	1.25	0.77
7	0.75	0.72
8	1.22	0.79
9	0.74	0.72
10	0.80	0.91

7. An assembly line has been producing 400 units of a product per day. Final inspection records show that on an average 20 units a day are defective. Construct a control chart for the % defective.

8. The mid-term and final exam scores of 10 students in a statistics course are recorded below.

a) Calculate the least squares regression line

b) Test the hypothesis that the slope is equal to 0 against the alternative that it is greater than 0 at the 5% significance level assuming normality for the error.

Midterm	Final		Midterm	Final
70	87		67	73
74	79		70	83
80	88		64	79
84	98		74	91
80	96		82	94

Solutions

1. We Under the null hypothesis, the mean $E[Y] = 192(0.75)$ and the variance $Var[Y]$=192(0.75) (0.25)

a) $P(Y \geq 152|p = 0.75) = P(Y \geq 151.5|p = 0.75) = P\left(Z \geq \frac{151.5-144}{6}\right) = P(Z \geq 1.25) = 0.1056$

b) Similarly, $\beta = P(Y < 152|p = 0.80) = 0.3524$

2. We compute

a) Construct the statistic

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}}$$

and reject whenever $|Z| > 1.96$

b) Here,$\hat{p}_1 = 0.037, \hat{p}_2 = 0.053$ and $|Z| = 1.726$. Hence we do not reject the null hypothesis

3. a) We must use the Student t distribution with 24 degrees of freedom since the variance is unknown. We compute the test statistic

$$t = \frac{308.8 - 500}{115.15/\sqrt{25}} = -8.30 < -2.492 = -t_{0.005}$$

Hence we reject the null hypothesis and accept the alternative that $\mu < 500$. That is the claim is accepted.

b) The p-value is the probability of observing a value smaller than 308.8. From the table it is smaller than 0.0005.

4. We compute the chi square statistic

$$\frac{(n-1)S^2}{\sigma^2} = \frac{22(147.82)}{100} = 32.52$$

The test rejects if the test statistic is less than 10.98 or greater than 36.78, values obtained from teh Chi square table when we have 22 degrees of freedom. Hence , the null hypothesis is not rejected.

5. The pooled estimate of common variance is equal to

$$\begin{aligned} s^2 &= \frac{(n_1-1)\,s_1^2+(n_2-1)\,s_2^2}{n_1+n_2-2} \\ &= \frac{(9)\,25+(14)\,16}{23} \\ &= 19.52 \end{aligned}$$

We must use the Student t distribution with 23 degrees of freedom. Note $\mathrm{t}_{0.025} = 2.069$. We may test the hypothesis by constructing a 95% confidence interval. It is given by

$$\bar{x}-\bar{y}\pm t_{0.025}s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$90-87\pm 2.069\,(4.4)\,\sqrt{\frac{1}{10}+\frac{1}{15}}$$

$$(-0.72, 6.72)$$

The interval contains 0 and so we have no evidence that the two types of cement differ at that level of confidence. So, we do not reject the null hypothesis.

6. This is an example on the paired t test. We compute the differences $d = X - Y$ for each lot and take the average $\bar{d}$ of the differences . We then have a one sample t test on the differences.

$$\frac{|\bar{d}|}{s/\sqrt{10}} = 1.477 < 2.262 = t_{0.025}$$

Hence, we do not reject the null hypothesis

7. This involves calculating the control limits

$$UCL = \bar{p} + 3\frac{\sqrt{\bar{p}\left(1-\bar{p}\right)}}{\sqrt{n}}, LCL = \bar{p} - 3\frac{\sqrt{\bar{p}\left(1-\bar{p}\right)}}{\sqrt{n}}$$

Here $\bar{p} = \frac{20}{400}, n = 400$

$$UCL = 0.08269, LCL = 0.0173$$

8. a) The regression line is

$$\hat{y} = 86.8 + \frac{842}{829}\left(x - 74.5\right)$$

b) the estimate of the slope is $\hat{\beta} = \frac{842}{829}$ and the estimate of variance is $s^2 = 17.9998$

To test the hypothesis that $\beta = 0$, we compute the t statistic and reject when

$$\sqrt{\sum\left(x_i - \bar{x}^2\right)}\left(\hat{\beta}\right)/s > 2.306$$